Chapter 9

Application of the Rasch Model for Testing Piaget's Theory of Cognitive Development

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The aim of the study was to investigate whether the assumption of unidimensionality that underlies the Piagetian model holds when applying the dichotomous Rasch model. The Piagetian model describes cognitive development as a progressive construction of new structures, appearing at approximately the same age in different behavioral domains. When testing this postulate, researchers usually administer two or more tasks from different domains to the same subjects, with the goal to determine whether levels of cognitive development are identical across domains (developmental synchronism). In practice, valid between-domain comparisons are extremely difficult to perform. The Piagetian model does not provide any criteria for the evaluation of parallelism of tasks (see de Ribaupierre, Rieben, & Lautrey, 1991; Wohlwill, 1973).

The goal of this study was to perform between-domain comparisons applying the dichotomous Rasch model (Rasch, 1960).

1. Method

Subjects. The study is part of the ongoing longitudinal project „Vienna Developmental Study“ which investigates conditions that facilitate or hinder development of cognitive competence and scholastic achievement. The data analyzed for this study are taken from the 93 subjects in the longitudinal project. Because of the small sample size for Rasch analyses additional data were taken from an independent sample of 42 early adolescents. The total sample consisted of 135 subjects (68 boys and 67 girls; mean age = 12.15 years; SD = .55 years).

Material. The two Piagetian tasks used in this study involved Syllogistic Deductions (verbal material; developed by Kodroff, & Roberge, 1975; revised by Edelstein, 1979) and Isolation of Variables (visual material; developed by Kuhn, & Brannock, 1977; extended by Edelstein, 1979) measuring the transition from the stage of concrete operations to the stage of formal operations. This transition normally takes place approximately at the age of twelve years (Piaget, 1950). Based on the results of a calibration study using an independent sample of 63 early adolescents, the two Piagetian tasks were revised.

Verbal Tasks (Syllogistic Deductions). In the present adaptation the Syllogistic Deductions involve two syllogistic statements, each presented in the four basic forms: affirmation of antecedent, negation of antecedent, affirmation of consequent, and negation of consequent. The first statement referred to a real life situation; the second statement was abstract. The syllogisms were presented verbally. First the experimenter read the premise and asked the

child to repeat it. If the child's repetition was wrong, the experimenter reread the premise and asked the child to repeat it again. Then the four syllogistic questions were presented. After the child had answered, the experimenter asked the child to give a reason for his answer. The questions were scored as „solved" if the answer was correct and an accurate explanation was given. In all other cases the questions were scored as „not solved". For „warming-up" an example was presented to the subjects. For material and instruction see Table 1.

Instruction:

I will read you some sentences which you are to remeber and then I will ask you a question to which you are to answer either yes, no or maybe.

Real life situation:

Premise: *If the sun is shining Michaela wears a red T-shirt.* (repetition)

Affirmation of antecedent: *The sun is shining. Does Michaela wear a red T-shirt?* (correct answer: yes)
Why? (explanation)
Negation of antecedent: *The sun is not shining. Does Michaela wear a red T-shirt?* (correct answer: maybe)
Why? (explanation)
Affirmation of consequent: *Michaela wears a red T-shirt. Is the sun shining?* (correct answer: maybe)
Why? (explanation)
Negation of consequent: *Michaela does not wear a red T-shirt. Is the sun shining?* (correct answer: no)
Why? (explanation)

Abstract situation:

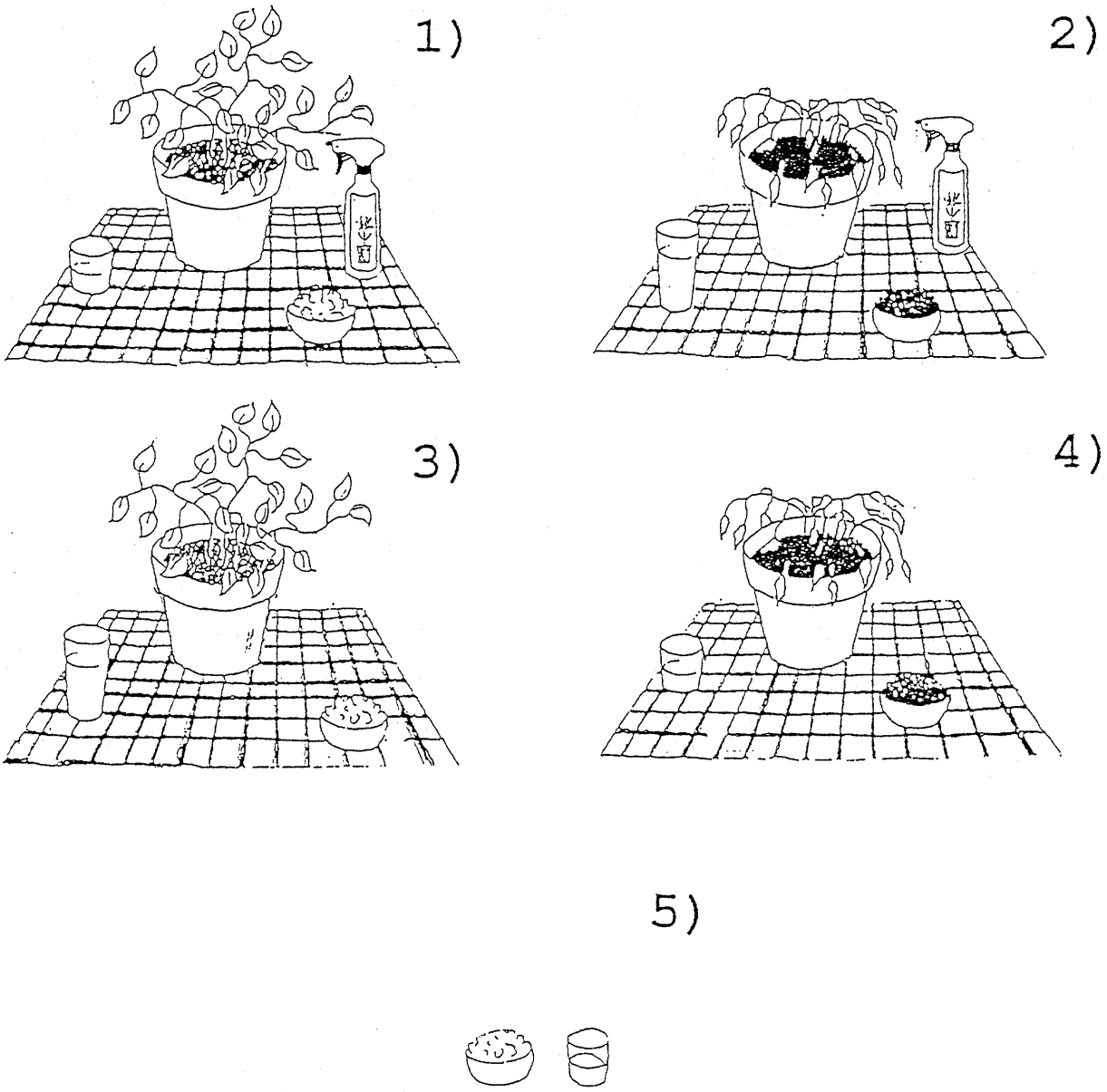
Premise: *If X goes to A, X travels by B.*

Negation of consequent: *X does not travel by B. Does X go to A?* (correct answer: no)
Affirmation of consequent: *X travels by B. Does X go to A?* (correct answer: maybe)
Affirmation of antecedent: *X goes to A. Does X travel by B?* (correct answer: yes)
Negation of antecedent: *X goes not to A. Does X travel by B?* (correct answer: maybe)

Table 1: Syllogistic deductions: Material and instruction

Visual Tasks (Isolation of Variables). The material for testing Isolation of Variables consisted of three real life problems (raising plants, getting presents, coating a roof). For each problem the subjects had to answer two questions, the first one concerning the isolation of the operative variable, and the second one concerning the exclusion of one of the inoperative variables (cf. Inhelder, & Piaget's physical problems, 1958).

For illustration the plant problem is described: Four plants were presented to the subjects. Two of the plants appeared quite healthy and two were clearly in poor condition. A glass of water, either large or small, and a dish containing either dark- or light-colored plant food were adjacent to each plant, and a bottle marked „leaf lotion" was adjacent to two of the plants too. The problem was constructed such that one variable (plant food) was operative in influencing the plant's health and the other two variables were ineffective. The subjects were asked to isolate the operative variable and to exclude inoperative variables. Both questions were scored as „solved" if the answer was correct and an accurate explanation was given. In all other cases the questions were scored as „not solved". For material and instruction see Figure 1.



Instruction:

I've been raising some plants. I'd like to show them to you and ask what you think. Let's look at this plant first (1). It seems quite healthy, doesn't it? Every week I gave this plant a large glass of water and some of this light-colored plant food.
Now look at this plant (2). It doesn't look so healthy, does it? Every week I gave this plant a large glass of water, some of this dark-colored plant food, and a little of the leaf lotion in this bottle.

The other two plants (3), (4) are similarly described.

Now I have another plant like this at home that I've just started working on. I'm giving my plant at home a small glass of water each week, some of the light-colored plant food (5), and I'm not giving it any of the leaf lotion.

Question 1: *How do you think my plant at home is going to turn out?* (correct answer: healthy)

How do you know? (explanation)

Question 2: *Does the leaf lotion have anything to do with how that plant turns out?* (correct answer: no)

How do you know? (explanation)

Figure 1: Plant problem: Material and instruction

Analyses. Application of the dichotomous Rasch model implies a so-called „empiristic" approach (remaining at the level of pass/fail scores) rather than a structuralist one (which is the typical approach when analyzing Piagetian tasks). Therefore, we measured performance in Syllogistic Deduction using eight dichotomous verbal items which resulted from combining two statements and four syllogistic forms. In an analogous fashion, we measured performance in Isolation of Variables using six dichotomous visual items which resulted from combining three real life problems and two tasks (isolation of the operative variable and exclusion of the inoperative variable). However, the answers to the visual tasks can not fulfil Rasch´s assumption of local independence. The answers regarding the exclusion of the inoperative variable are related to the answers regarding the isolation of the operative variable (see Figure 1). For that reason, only the responses to the first questions (isolation of the operative variable) were analyzed.

Some may argue that an empiristic approach leads to loss of information. However, application of the structuralistic approach causes problems as well. On the one hand, the number of verbal items is reduced to two because all reactions to each statement are needed for subjects' evaluation. On the other hand, reliability scores of such evaluations are usually low (see Kuhn, & Brannock, 1977).

Goodness-of-fit was tested via Andersen's (1973) conditional likelihood ratio test (CLRT) a commonly applied fit-statistic when studying unidimensionality across subgroups of subjects. Subjects were grouped regarding task performance, gender, and age. To investigate whether verbal items and visual items belong to the same latent dimension, the CLRT devised by Martin-Löf (1973) was applied. Whereas Andersen´s CLRT tests whether unidimensionality holds in different subgroups of *subjects*, Martin-Löf´s CLRT tests whether unidimensionality holds in different subgroups of *items.* The test statistic

$$Z = -2\ln\left\{\frac{L(\hat{\sigma})}{L(\hat{\sigma}_a)\cdot L(\hat{\sigma}_b)}\cdot\frac{\prod_{r=0}^{k}\left(\frac{n_r}{n}\right)^{n_r}}{\prod_{r_a=0}^{k_a}\prod_{r_b=0}^{k_b}\left(\frac{n_{r_a r_b}}{n}\right)^{n_{r_a r_b}}}\right\}$$

is asymptotically χ^2-distributed with $k_a \cdot k_b - 1$ degrees of freedom (Martin Löf, 1973; see also Kubinger, 1988). The following notation is used:

$L(\hat{\sigma})$	likelihood for the whole test with k items,
$L(\hat{\sigma}_a), L(\hat{\sigma}_b)$	likelihood for item group I_a with k_a items, likelihood for item group I_b with k_b items,
n_r	number of subjects with score r,
$n_{r_a r_b}$	number of subjects with score r_a in I_a and score r_b in I_b.

Because of the small sample size for Rasch analyses for all tests, the α-level was set to .05.

2. Results

Before applying CLRTs the distribution of item solution probabilities was examined. For all items acceptable values were observed (see Table 2).

ver1	ver2	ver3	ver4	ver5	ver6	ver7	ver8	vis1	Vis2	vis3
.704	.289	.259	.474	.526	.244	.600	.333	.356	.356	.237

Table 2: Item solution probalitites: ver.... verbal item; vis....visual item

Andersen´s CLRTs did not reject the assumption of unidimensionality in different subgroups of subjects. (Only five out of the 135 subjects had to be excluded from CLRTs because of bottom or ceiling effects.) However, Martin-Löf´s CLRTs showed that unidimensionality did not hold across domains (see Table 3).

	8 verbal items, 3 visual items		
Andersen's CLRT	χ^2	df	P
Task Performance	17.092	10	.0724
Gender	12.009	10	.2845
Age	17.950	10	.0558
Martin-Löf´s CLRT	51.063	23	.0007

Table 3: Check of unidimensionality across domains

For that reason, a second check was performed to investigate whether the assumption of unidimensionality holds within the domains. Andersen´s CLRT did not reject the assumption of unidimensionality for the verbal items ($\chi^2 = 5.737, df = 7, p = .5708$). Because of the small item number the CLRT could not be assessed for the visual items.

For theoretical reasons, the verbal items could be divided into subgroups of items in two ways: First, regarding the content (four concrete items and four abstract items, see Table 1); second, regarding the type of correct answers (four yes/no answers and four maybe answers; see Table 1). There is evidence suggesting that "yes/no" answers are easier for the subjects than "maybe" answers (Haars, & Mason, 1986). Results of Martin-Löf´s CLRTs rejected unidimensionality both across the content and across the types of answers (see Table 4).

Grouping	χ^2	df	P
Content of Items	27.745	15	.0232
Type of Answers	128.725	15	<.0001

Table 4: Check of unidimensionality within the verbal domain: Martin-Löf´s CLRT

From the Piagetian model (see Ribaupierre et al., 1991) one can derive that a subgroup of subjects might be advanced in cognitive development - within stage transition - while other subjects were still in the stage of concrete operations. Advanced subjects show item parameter vectors different from those of other subjects, e.g., they have a higher probability to solve difficult items (items with abstract content and items with "maybe" answers).

To test these assumptions we applied the Mixed Rasch model (MRM, Rost, 1990; Rost, & von Davier, 1992). Results showed that the 2-class MRM fit the data better than the unidimensional Rasch model (see Table 5).

Model	LR – χ^2	df	p	log Likelihood
Rasch Model	401.11	244	<.0001	-708.87
2 Classes MRM	253.12	234	.1863	-634.88
Comparison of the Models	148.00	10	<.0001	

Table 5: Check of Two-Classes MRM

57.8% of the subjects belonged to class 1, and 42.2% of the subjects belonged to class 2. Mean probability of class membership was .942 (SD = .103). For distribution of item difficulty parameters in the two classes see Figure 2. The two latent classes differed mostly in the four items with "maybe" answers. For subjects from class 1 „maybe" answers were much more difficult than for subjects from class 2.

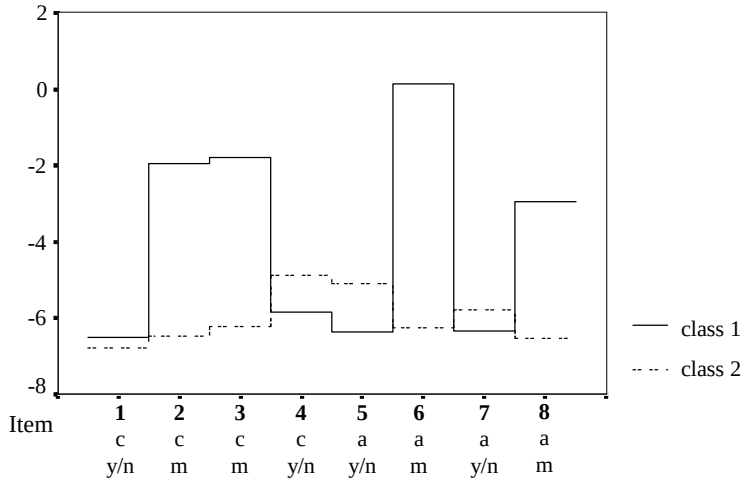


Figure 2: Distribution of item difficulty parameters in the two classes of the MRM:
c...concrete, a...abstract, y/n... correct answer is „yes" or „no", maybe ... correct answer is „maybe"

3. Discussion

This study investigated Piaget´s assumption of developmental synchronism using the dichotomous Rasch model. Verbal and visual items that measure the transition from the stage of concrete operations to the stage of formal operations were analyzed. If the assumption of developmental synchronism holds the items assess the same latent dimension regardless of the type of presentation (verbal versus visual). Data analyses using Andersen´s CLRT support this assumption. However, data analyses using Martin-Löf´s CLRT suggest that the verbal items and the visual items do not belong to the same latent dimension.

In addition, unidimensionality does not hold within the verbal domain when subgroups of items are compared. Therefore, Martin-Löf´s CLRT is recommended if theoretically based assumptions regarding subgroups of items are available. In general, application of different

types of CLRTs is a fruitful approach to investigate different theoretical assumptions (see also Gustafsson, 1980; Müller-Philipp, & Tarnai, 1988).

Application of the MRM to the verbal items supports the assumption that subjects belong to two populations that differ in cognitive development. This observation is in agreement with Piaget´s concept that the transition from the stage of concrete operations to the stage of formal operations takes place approximately at the age of twelve years. In addition, results suggest that a subgroup of items (items with „maybe“ answers) is particularly sensitive for measuring this transition from the concrete operatoric stage to the formal operatoric stage.

Additional analyses are suggested in two directions: First, an independent sample of subjects should be studied to validate the observations of the study; and second, an additional set of items should be developed to study whether a transfer of results is possible.

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