Chapter 12

Incorporating Item Response Theories in Causal Models

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1. Introduction

In this paper we will discuss how item response theories can be incorporated in causal models. In this way, we hope to link two important research traditions in the field of psychometric measurement models: the development of item response theories and the development of causal models with latent variables.

Item response theory occupies a prominent place in contemporary psychometric research. The development of adequate item response models for analyzing dichotomous or polytomous test or questionnaire data is a central topic in research on measurement models for the social and behavioral sciences. Item response models are based on rather strong assumptions about the functional relationship between scores on a (possibly multidimensional) latent continuum and the probability of a particular response to a dichotomous or polytomous item. Up till now research on item response models has mainly been concerned with developing efficient algorithms for estimating item and subject parameters. Due attention has also been given to the way in which item response models may help in solving (or, at least, in clarifying) many applied measurement problems such as the assessment of differential item functioning (item bias), or the problem of test equating.

Another important theme in the field of social and behavioral research methodology pertains to the formulation and testing of causal models that include latent variables. Models of this kind hypothesize an explicit causal ordering among the variables and are often represented by path diagrams in which directed arrows between two variables indicate the direction of the causal flow between them. If all variables are observable and are measured on a continuous scale, the path model can be translated in a system of regression equations, and the parameters of this system can be estimated by means of ordinary regression analysis. When some of the variables involved are not directly measurable but have to be measured via imprecise (and probably somewhat ad hoc) indicator variables, the structural part of the causal model, which represents the causal order among the manifest (i.e., directly observed) and latent (i.e., indirectly observed) variables, has to be augmented by a measurement model that specifies the relationship between a latent variable and its indicators. The measurement part of the model is usually taken as equivalent to a factor analytic model in which the continuous manifest variables are linear transformations (with unknown weights or factor loadings) of the continuous latent variables (the factors). This approach to causal modeling with latent variables is embodied in programs such as LISREL (developed by K. Jöreskog and co-workers, and for which an eighth generation version is already available) and EQS (developed by P. Bentler). Estimation of the unknown regression coefficients from the structural part of the model and the unknown factor loadings from the measurement part is usually based on the information provided by the means vector of the observed variables, and their variance-covariance matrix.

Problems with the linear structural equation approach to causal modeling arise when the observed variables are discrete. Research in the social and behavioral sciences seldom leads to measurements on a continuous interval scale, but only yields categorical variables for which the different response categories can, at most, be ordered from less to more. Although it is quite common to assign properly ordered numbers to the different response categories in such a way that a larger number represents a more pronounced or stronger response, treating these numbers as scores on a continuous interval scale is rather controversial, especially when these „scores“ are subsequently analyzed by rather „demanding“ programs such as LISREL or EQS. More specifically, it is quite dubious whether the measurement part of the causal model retains its validity when some of the observed variables are measured on a discrete ordinal rather than on a continuous interval scale.

The problems associated with the use of linear structural models for ordinal and especially dichotomous variables were duely recognized in the past. However, it is only quite recently that concentrated efforts have been made to adapt and extend linear structural modeling to the case of discrete indicator variables. By linking a generalization of Christoffersson's approach to factor analysis for dichotomous variables (Christoffersson, 1975) to the structural part of the basic LISREL model, Muthen (1984) arrived at a structural modeling approach that allows some or all of the latent variables to be measured by means of discrete ordinal indicator variables. More recently, Lee, Poon, and Bentler (1992) and Jöreskog (1993) proposed refinements of the Muthen methodology. A characteristic of this approach is that the model is fitted to the data using a two- or three-stage estimation procedure. In the first stage the unknown parameters from the measurement model are estimated, whereas in the next stage or stages the estimates of the parameters of the structural model are computed. Most often generalized least squares estimation is used. Arminger and Küsters (1988, 1989) developed a very general framework for the structural modeling of discrete and continuous data involving latent variables.

An alternative approach to causal modeling for discrete data is based on the logic of the latent class model. In this type of model both manifest and latent variables are treated as nominal scale variables and the causal relationships among all variables is described in terms of log linear models with latent variables. See Hagenaars (1990, 1993) for more details on and applications of this type of latent structure modeling. In psychology, however, most observed variables may be thought of as ordinal variables; the strict nominal treatment of the discrete variables, which is inherent in the latent class approach, may then not be entirely appropriate. An attempt to extend the latent class approach to include ordinal latent variables can be found in Croon (1990, 1991), whereas Heinen (1993) discussed the relation between discretized latent trait and latent class models.

In this paper we investigate the possibility of extending structural models with a measurement model based on item response theory. Work on this topic has been rather scarce. We may first refer to Erling Andersen's work on the estimation of characteristics of the latent distributions in the context of the Rasch model, e.g., Andersen & Madsen (1977) and Andersen (1980, 1985). A generalization of this approach can be found in Mislevy (1984, 1985) who described models in which explanatory variables may have an effect on the distribution of the latent variables in an item response model. Mislevy assumed that the item parameters were available from a prior calibration study. A similar model was thoroughly discussed and applied by Verhelst and Eggen (1989).

In this paper we extend the approach proposed by Mislevy in two ways. First, we will formulate models in which the latent traits themselves may have further causal effects on observed variables; and, second, we do not assume that the item parameters are known in advance.

In the next section we discuss the general framework for the type of models we have in mind. In section 3 we discuss a particular example of a causal model with one latent trait. In section 4 we discuss the results of some analyses using our algorithm for this particular model on generated and real data.

2. The general framework

The general framework within which we will discuss the incorporation of latent traits in causal models is based on Arminger and Küsters (1988, 1989). Following these authors, we assume that the model contains exogeneous variables whose association structure is determined by factors outside the model. These exogeneous variables may, however, exert causal influences on both a set of latent variables and a set of observed endogeneous variables. Moreover, the latent variables themselves may have causal effects on these endogeneous variables. Finally, the measurement part of the causal model specifies how the latent variables are measured by a set of manifest indicator variables. It is also interesting to spell out explicitly which causal effects are assumed to be absent from the model. First, we do not assume that the exogeneous variables have a direct effect on the indicator variables; second, we do not assume a direct causal relationship between the indicator variables and the endogoneous variables.

Let vector Y represent the p manifest exogeneous variables which may exert a causal influence on the q latent variables represented by vector θ and the r manifest endogenous variables represented by vector Z. The latent variables θ are measured via a set of m indicator variables X. The following path diagram represents the structural and the measurement part of the causal model that links the four sets of variables (see also chapter 1.9):

$$\begin{array}{ccccc} Y & \rightarrow & \theta & \rightarrow & X \\ & & \downarrow & & \\ & & Z & & \end{array}$$

Figure 1: Path diagram of the latent trait causal model

In a causal diagram of this type a directed arrow $(U \rightarrow V)$ from variable U to variable V indicates that U has an effect on V, which implies that the conditional distribution of V depends on U. Note that there is no direct effect of Y on X: the association between the exogeneous variables and the indicator variables is completely explained by the intervening latent variables θ. Furthermore, the indicator variables X have no direct effect on the endogeneous variables Z: any association or correlation between these two sets of variables is considered as spuriously caused by the common „third" variables θ. Hence, for fixed values of θ, X is independent of both Y and Z.

The preceding diagram implies the following decomposition of the joint distribution of (Y, θ, X, Z):

$$f(Y,\theta,X,Z) = f(Y)\, f(\theta|Y,\alpha)\, p(X|\theta,\beta)\, f(Z|Y,\theta,\gamma). \tag{1}$$

The distribution f(Y) is the marginal distribution of Y and is of minor importance in our approach. If this distribution contains unknown parameters, they may be estimated independently of the parameters occurring in the other distributions. The structural part of the model is represented by the following two conditional distributions. First, we have the conditional distribution $f(\theta|Y,\alpha)$ of the latent variables θ given the manifest variables Y, which is a function of the unknown parameters α. Second, we have the conditional distribution $f(Z|Y,\theta,\gamma)$ of the endogeneous variables Z given the latent variables θ and the exogeneous variables Y, which is a function of the parameters γ. The measurement part of the model is given by the conditional distribution $p(X|\theta,\beta)$, which is function of the unknown parameters β. In the context of an item response theory, β represents the item parameters.

Since the observations on the latent θ are missing, the EM-algorithm can be used to estimate the unknown parameters α, β and γ. Application of this algorithm requires the determination of the conditional distribution $h(\theta|Y,X,Z)$ of θ given (Y,X,Z). It is easy to see that

$$\begin{aligned} h(\theta|Y,X,Z) &= \frac{f(Y,\theta,X,Z)}{\int f(Y,\theta,X,Z)\,d\theta} \\ &= \frac{f(\theta|Y,\alpha)\,p(X|\theta,\beta)\,f(Z|Y,\gamma)}{\int f(\theta|Y,\alpha)\,p(X|\theta,\beta)\,f(Z|Y,\theta,\gamma)\,d\theta}. \end{aligned} \tag{2}$$

Let $h_0(\theta\,|\,Y,X,Z)$ represent the conditional distribution evaluated for provisory estimates of the unknown parameters. Then, new estimates of these parameters can be obtained by maximizing the expected value of the log likelihood of the complete data:

$$E \ln f(Y,\theta,X,Z) = \int \ln f(Y,\theta,X,Z)\, h_0(\theta|Y,X,Z)\, d\theta. \tag{3}$$

If the three sets of parameters are disjoint, this optimization problem breaks down in three separate subproblems as a result of the particular decomposition of the joint distribution of (Y,θ,X,Z):

1. Determine new estimates of α by maximizing

$$E \ln f(\theta|Y,\alpha) = \int \ln f(\theta|Y,\alpha)\, h_0(\theta|Y,X,Z)\, d\theta \ . \tag{4}$$

2. Determine new estimates of β by maximizing

$$E \ln p(X\,|\,\theta,\beta) = \int \ln p(X\,|\,\theta,\beta)\, h_0(\theta\,|Y,X,Z)\, d\theta \ . \tag{5}$$

3. Determine new estimates of γ by maximizing

$$E \ln f(Z|Y,\theta,\gamma) = \int \ln f(Z|Y,\theta,\gamma)\, h_0(\theta\,|Y,X,Z)\, d\theta \ . \tag{6}$$

The step in which the item parameters β are estimated simplifies considerably if each latent variable θ_m is measured by a specific set of indicator variables X_m which are independent of the other latent variables. If β_m denotes the set of parameters that describes the relationship between X_m and θ_m, one may write

$$p\left(X\middle|\,\theta,\beta\right)=\prod_{m} p\left(X_m\middle|\,\theta_m,\beta_m\right) \ . \tag{7}$$

Then, each set of parameters β_m can be estimated separately by maximizing

$$\int \ln p\left(X_m\middle|\,\theta_m,\beta_m\right) h_0\left(\theta_m\middle|Y,X,Z\right) d\theta_m \ , \tag{8}$$

in which $h_0(\theta_m)$ is the marginal distribution of θ_m obtained by integrating $h_0(\theta)$ over all latent variables θ_k: $\neq$ m. Note that the conditional distribution h_0 itself does not necessarily factorize as a product of conditional distributions for the separate components θ_m. Such a factorization is possible when the distributions $f\left(\theta\middle|Y,\alpha\right)$ and $f\left(Z\middle|Y,\theta,\gamma\right)$ also factorize in the following way:

$$f\left(\theta\middle|Y,\alpha\right)=\prod_{m} f\left(\theta_m\middle|Y,\alpha_m\right) \tag{9}$$

and

$$f\left(Z\middle|Y,\theta,\gamma\right)=\prod_{m} f\left(z_m\middle|Y,\theta_m,\gamma_m\right) \ , \tag{10}$$

i.e., when, first, the different latent variables are conditionally independent, and, second, the set of endogenous variables can be partitioned in different subsets with each subset being under the influence of exactly one latent variable.

A further simplification occurs when the relationship between the indicator variables X_m and the latent variables θ_m satisfies the Rasch model for dichotomous items or one of its extensions for polytomous data. In this case sufficient statistics s_m exist for the item parameters β_m and the relevant part of the likelihood function can be further decomposed as

$$p\left(X_m\middle|\,\theta_m,\beta_m\right)=p_1\left(X_m\middle|s_m,\beta_m\right)p_2\left(s_m\middle|\,\theta_m,\beta_m\right) \ . \tag{11}$$

Note that factor p_1 is independent of θ_m. Maximizing p_1 with respect to β_m gives the conditional maximum likelihood estimates of these parameters, and these estimates may be obtained prior to the fitting of the complete causal model. The conditional estimates of β_m may then be used to evaluate the second factor p_2 throughout the entire iterative process during which now only the parameters pertaining to the structural part of the causal model have to be estimated. It should be realized, however, that this „conditional“ approach to causal modeling based on the Rasch model will not necessarily give the same results as the „marginal“ approach in which item parameters and structural parameters are estimated simultaneously. For the simple case of the isolated univariate Rasch model, De Leeuw and Verhelst (1986) and Lindsay, Clogg and Grego (1991) have shown that the conditional and marginal item parameters are identical only if the observed total-score distribution satisfies some rather stringent inequality constraints. When these constraints are violated, the conditional and marginal estimates will differ; moreover, given the conditional estimates, it then proves to be impossible to find a latent distribution for which the observed total-score distribution is exactly reproduced. Without any doubt, similar considerations apply when the Rasch model is incorporated into a causal model.

In section 3 the general approach will be illustrated by a specific example in which only one latent variable θ is involved.

3. Incorporating the two-parameter logistic model for one latent trait

In this section we make some more specific assumptions in order to obtain a workable model in which a single latent trait, measured by dichotomous indicator variables that satisfy a two-parameter logistic model, is incorporated in a fulfledged causal model for analyzing data obtained in a sample of N respondents. An arbitrary respondent will be denoted by the subscript v.

First, we assume that the conditional distribution of θ_v is normal with expected value $\theta_v^* = \sum_{j=1}^{p} \alpha_j Y_v$ and constant error variance ω_1^2. Using vector notation we may also write $\theta_v^* = Y_v' \alpha$ in which Y_v' represents the vector of observed scores of respondent v on all p exogeneous variables. The vector α of order $p \times 1$ represents the unknown regression parameters. In order to include a constant coefficient in the regression equation for θ, a constant variable with all scores equal to 1 should be added to the set of exogeneous variables. For this part of the model, each individual respondent's contribution to the likelihood function is:

$$F_{v1} = \frac{1}{\omega_1 \sqrt{2\pi}} \exp\left[-0.5.\left(\frac{\theta_v - Y_v' \alpha}{\omega_1} \right)^2 \right] . \qquad (12)$$

Assuming that all θ_v's are independently distributed, the first factor of the likelihood functions is simply $F_1 = \prod_v F_{v1}$.

The second part of the structural model concerns the conditional distribution of Z given Y and θ. Here, we make the assumption that Z_v given (Y_v, θ_v) follows a normal distribution with expected values Z_{vk}^* given by

$$Z_{vk}^* = \sum_{j=1}^{p} \delta_{jk} Y_{vj} + \zeta_k \theta_v \qquad (13)$$

and constant conditional variance/covariance matrix Ω_2 of order $r \times r$. In this expression, δ_{jk} is the regression coefficient of exogeneous variable Y_j from the equation for the endogeneous variable Z_k; the parameter ζ_k is the regression coefficient of the latent variable θ in the same equation. In this system of regression equations a constant variable should also be included in the set of independent variables in order to guarantee that the regression equation contains a constant coefficient. For this second part of the model, a respondent's contribution to the likelihood function can be written as

$$F_{v2} = \frac{1}{(2\pi)^{p/2} \left| \Omega_2 \right|^{1/2}} \exp\left[-0.5 \left(Z_v - Z_v^* \right)' \Omega_2^{-1} \left(Z_v - Z_v^* \right) \right] . \qquad (14)$$

The overall contribution of this second part is $F_2 = \prod_v F_{v2}$.

As the measurement part of the model we take the well-known item response model for dichotomous items proposed by A. Birnbaum in Lord & Novick (1968). Under this model the probability of a positive response by subject v to item i is given by

$$p_{vi}(\theta_v) = p(X_{vi} = 1 | \theta_v) = \frac{e^{\beta_i(\theta_v - \sigma_i)}}{1 + e^{\beta_i(\theta_v - \sigma_i)}}, \quad (15)$$

in which θ_v represents the subject's value on the latent continuum (see eq. (10) in chapter 1.3). The parameter β_i is the item discrimination parameter which determines the steepness of the item characteristic curve: the larger β_i is the faster $p_{vi}(\theta)$ changes as a function of θ. The parameter σ_i is a location parameter. Under the assumption of local independence, each respondent's contribution to the likelihood function for the measurement part of the model is

$$F_{v3} = \prod_i \frac{e^{x_{vi}\beta_i(\theta_v - \sigma_i)}}{1 + e^{\beta_i(\theta_v - \sigma_i)}}. \quad (16)$$

The overall contribution of the measurement part is then $F_3 = \prod_v F_{v3}$. In this paper the item parameters β_i and σ_i are assumed to be unknown.

The causal model described above is not yet identified since the scale of the latent variable θ is not fixed. Any linear transformation of θ can be compensated by an appropriate rescaling of the model parameters. In order to solve this indeterminacy problem, the item parameters were normalized so that $\Sigma_i \sigma_i = 0$, and $\omega_1 = 1$. In this way the arbitrary origin and unit of the θ-scale was fixed. Note that under these identification constraints, the conditional error variance/covariance matrix Ω_2 is completely identified.

The distribution of the observed variables Y_v, X_v, and Z_v can be obtained by integrating θ out of the product $F_{v1}\, F_{v2}\, F_{v3}$:

$$f(X_v, Y_v, Z_v) = \int F_{v1}\, F_{v2}\, F_{v3}\, d\theta. \quad (17)$$

The maximum likelihood estimates of both the regression coefficients from the regression equations and the item parameters are obtained by maximizing

$$\phi = \sum_v \ln f(X_v, Y_v, Z_v). \quad (18)$$

Although in principle this optimization problem could be solved by a Newton-Raphson procedure, we preferred to adopt the EM algorithm by treating the latent scores θ_v as missing data. More technical details on the iterative optimization procedure can be found in Croon and Heinen (1995). This procedure yields parameter estimates that maximize the log likelihood function.

Let the maximum value of ϕ be denoted by L. Now suppose we want to compare two different causal models with one model being a submodel of the other one. This hierarchical relation between two models may arise by setting some parameters in the more general model equal to a fixed constant, or by imposing equality constraints on some of the parameters. Let L_1 and L_2 denote the maxima of the log likelihood functions under the general and the restricted model. Then, standard statistical theory yields that the test statistic $-2(L_1 - L_2)$ is asymptotically chi-square distributed with degrees of freedom equal to the difference between the number of parameters in both models. This test procedure can be used to test whether the restricted model results in a significantly worse fit than the more general model. As such, tests of this kind may be helpful in selecting the most parsimonious set of structural relations among the variables in the causal model, and in selecting the most appropriate measurement model.

4. Application to data from a Social Security survey

In this section, the structural model developed above is applied to a real data set obtained from a large-scale survey. In the multidisciplinary research project *Social Security: Research on Demographic and Psychological Aspects*, sponsored by the Dutch Ministry of Education, several social and psychological effects of being unemployed were investigated by means of a large scale survey. In this section we analyze data which relate *Gender* and *Labor Market Status* with *Perceived Financial Situation* and *Meaningfulness of Life*. As exogeneous variables we defined two dummy variables Y_1 and Y_2:

- Y_1 represents *Gender* with $Y_1 = 0$ for women and $Y_1 = 1$ for men, and
- Y_2 represents *Labor Market Status* with $Y_2 = 0$ for unemployed and $Y_2 = 1$ for employed.

Furthermore, in order to estimate a possible interaction effect for *Gender* and *Labor Market Status*, we defined a third dummy variable $Y_3 = Y_1 \times Y_2$.

Subjects were asked to evaluate their present financial situation on four questionnaire items which originally had five response categories. Since one of these items proved to have very small intercorrelations with the other four, we decided to retain the four most homogeneous items. After dichotomization these four items were considered to be indicators $(X_1,\dots,X_4)$for a latent variable *Perceived Financial Situation* (θ). The four items, translated almost literally, were:

1. I have a good income.
2. I can afford things financially.
3. I don't have to make debts.
4. I can save some money.

The endogenous variable *Meaningfulness of Life* (Z) was defined on the basis of the respondents' reactions to 12 bipolar semantic differential scales on which they had to rate the quality of their present life. Some examples of these scales were: boring versus interesting, pleasant versus wretched, sure versus unsure, etc. Since the semantic differentials had seven response categeories, the sum of the 12 scales ran from 12 to 84. In the present analysis this sum variable, which was standardized before entering the analysis, is treated as the continuous endogeneous variable.

In the sequel we report the results of several analyses in which several hypotheses about the structure of both regression equations were tested. In all cases sample size was N = 563. In the first analysis θ was regressed on $\{Y_1,Y_2,Y_3\}$ and Z on $\{Y_1,Y_2,Y_3,\theta\}$. The value of the log likelihood function was $L_1 = -555.8386$.

Since the regression coefficients of Y_3 in both regression equations were very small, we performed a second analysis in which θ was regressed on $\{Y_1,Y_2\}$ and Z on $\{Y_1,Y_2,\theta\}$. Now, $L_2 = -555.9180$ and $-2(L_2 - L_1) = 0.1588$; the value of the last quantity is very small, given the fact that the constrained analysis contains two regression coefficients less. The conclusion that the interaction between *Gender* and *Labor Market Status* can be eliminated from both regression equation seems to be warranted.

In the third analysis we investigated whether Y_1 and Y_2 can be removed from the regression analysis of Z. Hence, θ was regressed on $\{Y_1, Y_2\}$ but in the second regression equation Z was only regressed on θ. This resulted in $L_3 = -556.7985$ with $-2(L_3 - L_2) = 1.7610$, a difference which still is acceptable given that two regression coefficients less were estimated. In a final step we checked whether the first regression equation could be further simplified by removing some of the exogeneous variables from it without a substantial loss in model fit. This proved to be impossible. For instance, removing the weakest exogeneous variable Y_1 from the regression equation for θ (and regressing Z itself only on θ) resulted in $L_4 = -567.4481$ with $-2(L_4 - L_3) = 21.2992$ which is much too large compared to the one regression coefficient which is removed from the model.

Thus, it seemed reasonable to accept the third analysis as the best one. The estimated regression equations were:

$$\hat{\theta} = -1.31 + 0.57\,Y_1 + 2.09\,Y_2 \quad \text{and} \quad \hat{Z} = -0.09 + 0.23\,\theta \quad \text{with} \quad \omega_2 = 0.6355\ .$$

This analysis confirms that *Gender* and *Labor Market Status* both have a clear effect on the latent variable *Perceived Financial Situation*. Since both independent variables are defined as 0/1 dummies, their regression coefficients are directly comparable. This comparison shows that, as might be expected, *Labor Market Status* has the largest partial effect on θ: holding *Gender* constant, employed people evaluate their financial status more favorably than unemployed people. The partial effect of *Gender* shows that, holding *Labor Market Status* constant, men tend to have a larger average score on θ than women. The most interesting feature of this analysis is probably shown by the second regression analysis. Neither of the two exogenous variables *Gender* and *Labor Market Status* has a direct effect on the endogenous variable Z; they have only indirect effects via the latent variable θ. The association between the exogenous and endogenous variables is completely explained by the intervening latent variable θ.

The estimates of the item parameters were:

i	β_i	σ_i
1	2.466	–0.070
2	4.256	–0.009
3	0.658	–1.543
4	2.239	0.540

From the latter table we may infer that the items have markedly different discriminatory power. Moreover, the same structural model was fitted to the data under the assumption that the manifest indicators for the latent variable had equal discrimination parameters, i.e., the measurement model was assumed to be a one-parameter Rasch model. However, the fit of this model was very bad: $L_5 = -637.0042$ so that $-2(L_3 - L_5) = 160.4114$, a value which compares very unfavorably with the number of free parameters lost in fitting the Rasch model.

References

Andersen, E. (1980) Comparing latent distributions. *Psychometrika, 45*, 121-134.

Andersen, E. (1985) Estimating latent correlations between repeated testings. *Psychometrika, 50,* 3-16.

Andersen, E. & Madsen, M. (1977) Estimating the parameters of the latent population distribution. *Psychometrika, 42*, 357-374.

Arminger, G. & Küsters, U. (1988) Latent trait models with indicators of mixed measurement level. In: R. Langeheine & J. Rost (Eds). *Latent Trait and Latent Class Models.* New York: Plenum, pp. 51-73.

Arminger, G. & Küsters, U. (1989) Construction principles for latent trait models. In: C.C.Clogg (Ed). *Sociological Methodology. Volume 19.* Washington: American Sociological Association, pp. 369-393.

Christoffersson, A. (1975) Factor analysis of dichotomized variables. *Psychometrika, 40,* 5-32.

Croon, M. (1990) Latent class analysis with ordered latent classes. *British Journal of Mathematical and Statistical Psychology, 43*, 171-192.

Croon, M. (1991) Investigating Mokken scalability of dichotomous items by means of ordinal latent class analysis. *British Journal of Mathematical and Statistical Psychology, 44,* 315-331.

Croon, M. & Heinen, T. (1995) Item response theories and causal modelling: A general framework. Tilburg: WORC Research Paper 95.12.26/7.

De Leeuw, J. & Verhelst, N. (1986) Maximum likelihood estimation in generalized Rasch models. *Journal of Educational Statistics, 11*, 183-196.

Hagenaars, J.A. (1990) *Categorical Longitudinal Data: Loglinear Panel, Trend, and Cohort Analysis.* Newbury Park: Sage.

Hagenaars, J.A. (1993) *Loglinear Models with Latent Variables.* Newbury Park: Sage.

Heinen, A.G. (1993) *Discrete Latent Variable Models.* Tilburg: Tilburg University Press.

Joreskog, K. (1993) Latent variable modeling with ordinal variables. In: K. Haagen, D.J. Bartholomew & M. Deistler (Eds). *Statistical Modelling and Latent Variables.* Amsterdam: North-Holland, pp. 163-171.

Lee, S.-Y., Poon, W.-Y. & Bentler, P.M. (1992) Structural equation models with continuous and polytomous variables. *Psychometrika, 57*, 89-105.

Lord, F.M. & Novick, M.R. (1968) *Theory of Mental Test Scores*. Reading: Addison-Wesley.

Lindsay, B., Clogg, C.C. & Grego, J. (1991) Semiparametric estimation in the Rasch model and related models, including a simple latent class model for item analysis. *Journal of the American Statistical Association, 86*, 96-107.

Mislevy, R.J. (1984) Estimating latent distributions. *Psychometrika, 49*, 359-381.

Mislevy, R.J. (1985) Estimation of latent group effect. *Journal of the American Statistical Association, 80*, 359-381.

Muthen, B. (1984) A general structural equation model with dichotomous, ordered categorical, and continuous latent variable indicators. *Psychometrika, 49,* 115-132.

Tsutakawa, R.K. (1984) Estimation of two-parameter logistic item response curves. *Journal of Educational Statistics, 4*, 263-276.

Verhelst, N.D. & Eggen, Th.J.H.M. (1989) *Psychometrische en Statistische Aspecten van Peilingsonderzoek. (PPON-rapport nr. 4).* Arnhem: Cito.