Chapter 25

Comparative Social Research with Multi-Sample Latent Class Models

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1. Introduction

Latent class analysis provides a powerful, flexible approach to the analysis of categorically-scored data. Perhaps the most widespread use of the latent class model (LCM) is as a categorical data analog to factor analysis. For example, LCMs are often used as a data reduction technique to examine the latent structure of a joint distribution of a set of indicator items, often with an interest in „building clusters for qualitative data“ (Formann 1985, p.87) or as an explicit scaling and measurement model to investigate the relations between the categorical indicators and the underlying categorical latent variable(s) which the indicators are intended to measure (see e.g., Clogg and Sawyer, 1981; Croon, 1990; Heinen, 1996).

When researchers wish to analyze data from more than a single population at one point in time, LCMs may be used to analyze data from several samples simultaneously. The groups may be different nations, states, regions, or cultural groups, when the research interest is in the context of cross-cultural comparisons, or, when the research focusses on social change, separate samples drawn from the same population at two or more time points (see e.g., McCutcheon, 1987; Hagenaars, 1990, chapters 3 and 6). Indeed, the groups may be any mutually exclusive set of observations on which identical variables are measured. For example, we consider the case where identical measures have been collected for samples of the American and Dutch publics. In Table 1 are the results of two separate latent class analyses of the respondents' attitudes toward protestors and legal demonstrations in the summer of 1974 for these two nations. The data are from the Political Action Panel Study (Barnes and Kaase, 1979; Jennings, van Deth, et al. 1991), and assess whether the respondents A) approve or disapprove of legal demonstrations, B) would or would not participate in such demonstrations, C) oppose or support a law to ban public demonstrations, and D) feel positively or negatively toward student demonstrators.

Generally, the results reported in Table 1 appear to suggest substantial similarity in the latent structure of American and Dutch attitudes toward public protest. However, several more specific questions might be raised with regard to the comparability of the results. We note, for instance, that in both samples Class I respondents appear more likely than Class II respondents to respond more positively towards student demonstrators. However, the conditional probability of expressing positive feelings on item D is higher in the Netherlands than in the USA, both for latent class I and latent class II. Should we give any substantive interpretation to this finding or should it be regarded as a difference due to sampling fluctuation? And if it turns out to be a real difference, what are the consequences for the interpretation of the latent variable. Further, as a final example of questions to be asked, given an appropriate interpretation of the underlying latent variable, may we conclude from the outcomes that there are more people in the Netherlands in favor of protests (.845) than in the USA (0.725), or is this difference also due to sampling?

	Dutch		*American*	
Indicator Item	*Class I*	*Class II*	*Class I*	*Class II*
A. Approval (Approve)	.976	.239	.965	.229
B. Participation (Would)	.872	.152	.944	.242
C. Law (Oppose)	.961	.531	.923	.351
D. Student Demonstrators (Positive)	.714	.358	.431	.144
Latent Class Probabilities	.845	.155	.725	.275

Table 1: Conditional and latent class probabilities for support of public demonstrations: 1972 american and dutch samples

In this chapter, we examine many of the issues involved in the estimation and interpretation of such multi-sample LCMs, and discuss their applications in comparative research. We first present the modifications of the LCM that allow for the simultaneous estimation of LCMs in two or more populations by including a *grouping* variable. We also show that the inclusion of a grouping variable highlights the differential emphasis of the two basic parameterizations of the LCM: the probabilistic and loglinear parameterizations. Beginning with a brief review of the probabilistic parameterization of the multi-sample LCM introduced by Clogg and Goodman (1984, 1985, 1986), we then go on showing how the loglinear parameterization can be used in an analogous way, and introduce some relevant models that are often overlooked. Next, we show that, although these two parameterizations are equivalent (Hagenaars, 1990; Heinen, 1993), they lead to different ways of looking at the LCM, and at the problems of comparative research. Thus, it is worthwhile to consider them both. Finally, we present a detailed consideration of our empirical example of multi-sample LCMs.

2. Multi-Sample Latent Class Models

The basic LCM includes only two types of variables: indicator and latent variables. Indicator variables are categorically-scored, manifest (observed) variables that are directly measured; their joint distribution provides the empirical basis for the latent class analysis. The latent variable is an unobserved, categorical variable. In the joint distribution of the indicator and latent variables, indicator variables exhibit direct, partial associations only with the latent variable(s); the conditional independence among the indicator variables when controlling for the latent variable is required by the axiom of local independence (Lazarsfeld and Henry, 1968; for an exception, see Hagenaars, 1988b).

The multi-sample case arises when a researcher has data from two or more populations (or two independent samples from the same population at two or more points in time) in which the identical set of indicator variables is included.[1]

Consider, for example, a typical situation from cross-national research in which we have four indicator items (k = 4) observed in two (or more) national surveys (S = 2), such as in our American and Dutch example. We can think of this as two (or more) 4-variable joint

[1] It is also to examine cases in which a core set of indicator items are present in two (or more) samples, and where some items are present in one sample but not the others(s). Although not explicitly discussed here, these mulit-sample methods can be routinely extended to include such cases of missing data (see eg., Fuchs, 1982, Fay, 1986; Hagenaars, 1988a)

distributions or as a *single 5-variable* joint distribution where the final variable is the two (or more) level grouping variable for nation.

Thus, in addition to indicator and latent variables, the multi-sample LCM includes a third type of variable: the *grouping* variable. The grouping variable is a categorically-scored, manifest variable that is allowed to exhibit associations with indicator and latent variables; that is, grouping variables are not constrained by the axiom of local independence. It is important to note that, in principle, grouping variables are categorically-scored variables; covariates, whose level of measurement are typically scored as continuous, are less relevant to comparative studies, and will not be considered here. Readers interested in LCMs with continuous covariates, or concomitant variables, may wish to examine work by Dayton and Macready (1988a, 1988b), van der Heijden, Mooijaart, and de Leeuw (1992), Formann (1992), McCutcheon (1994), and Croon and Heinen (chapter 12).

While we will focus primarily on multi-sample LCMs for comparative research, and thus on the inclusion of only a single grouping variable (e.g., nation), we should note that it is possible to include more than one grouping variable in LCMs. These additional categorical variables may act as either causes or consequences of the latent variable(s). Thus it is easy to extend the models presented here to 'modified LISREL' models (Hagenaars, 1993).

2.1 Probabilistic Parameterization of the Multi-Sample LCM

Consider the case in which a researcher has four indicators collected in two independent samples, e.g., each sample drawn from a different nation, and wishes to apply a latent class analysis employing one latent variable to both data sets. Throughout this paper, we consider the case with four indicator variables, a single 2-level grouping variable, and a single latent variable. The models can, of course, be readily generalized to accommodate more (or fewer) indicator variables, as well as more than a single latent variable, more than two samples, and more than a single grouping variable.

In its probabilistic parameterization, Clogg and Goodman (1984, 1985, 1986) refer to this as the *simultaneous* LCM, and it is clear that this model is a direct extension of the basic latent class model (36), chapter 1:

$$p\left(\mathbf{x}_{v|s}\right) = \sum_{g=1}^{G} \pi_{g|s} \prod_{i=1}^{k} \pi_{ix|gs} \qquad (1)$$

A comparison of (1) and (36) indicates that we must now include the grouping variable as one of the elements of the multi-sample LCM. Here, $\pi_{g|s}$ 1represents the conditional probability of an observation being in latent class g given that it is in sample s; $\pi_{ix|gs}$ 2represents the conditional probability that an observation from sample s and class g will be at level x of indicator i. Like (36), the multi-sample model is subject to a series of restrictions to permit identifiability. The conditional probabilities are restricted as $\sum_{x=0}^{m} \pi_{ix|gs} = 1.0$ 3and the latent class probabilities are also restricted as $\sum_{g} \pi_{g|s} = 1.0$.4

In comparative social research, our first concern usually focuses on the issue of *model invariance* or comparability. Thus, in this parameterization we first explore whether the models for the several samples are similar with respect to the number of classes in the latent variable and the pattern of relations between the indicator and latent variables. We then explore whether the model parameters are identical by imposing equality constraints on the conditional probabilities. In the probabilistic parameterization of the multi-sample LCM, model invariance can be separated into measurement homogeneity and distributional homogeneity. Measurement homogeneity focuses on the question of whether the same set of conditional probabilities characterize the latent classes in each of the *S* populations. Once the issues of measurement homogeneity are resolved, the researcher may turn to an examination of distributional homogeneity which focuses on the degree to which the same (conditional) latent class probabilities characterize the latent class distribution in the *S* populations (e.g., $\pi_{g|1} = \pi_{g|2}$).

The first measurement homogeneity hypothesis examines the form of the LCM in the *S* groups, and, given our restriction to one latent variable models, focuses on whether a similar number of latent classes is required to characterize the latent structure of each of the groups (i.e., whether $G_1 = G_2 = \ldots G_S$). This step is somewhat analogous to testing the factor invariance hypothesis in the continuous data model (see e.g., Marsh et al., 1985; Byrne et al., 1989), in which the analyst first considers the required number of factors for each group (although in most comparative latent class analyses, the possibility of more than one latent variable also must be seriously considered, see Hagenaars 1990, section 3.6). It is possible, for example, that the latent structure in one group requires more latent classes than in another group in order to find an acceptable model fit. In a case where there are more latent classes in the LCM for one group than for another (e.g., $G_1 > G_2$), we have clear evidence that the model lacks complete invariance across the two groups and we must look for explanations of this invariance, partly making use of the parameter estimates for the two samples.

If the number of latent classes is the same in both groups, there is less measurement invariance, but still not necessarily complete *invariance*. For the latent classes in the two samples to be the same (to have the same meaning), we usually require that the pattern of conditional response probabilities belonging to a particular latent class g is similar in both groups, or even stronger: that they have the same values in both groups:

$$\pi_{ix|g1} = \pi_{ix|g2}\,.$$

Assuming that the model without these restrictions provides an acceptable fit, we test the acceptability of these restrictions by obtaining the likelihood-ratio chi-squares (L^2) for the model with these restrictions; the difference between the two model L^2s provides a conditional test that the four indicator variables are invariantly related to Class g in groups 1 and 2. The degrees of freedom for the conditional L^2 test are obtained as the difference in degrees of freedom for the two models.

Given that the similarities and differences of the conditional response probabilities between the two groups lend themselves to a sound theoretical interpretation and lead to the conclusion that the latent variable can be given the same substantive interpretation in both groups, it makes sense to proceed to what is usually the final hypothesis in the hypothesis hierarchy: the distributional homogeneity hypothesis, by imposing an across-group equality constraint on the respective (conditional) latent class probabilities of the two groups populations (e.g., $\pi_{g|1} = \pi_{g|2}$).

2.2 Loglinear Parameterization of the Multi-Sample LCM

In this section we examine the multi-sample LCM in the loglinear parameterization suggested by Haberman (1979). Once again, we consider the case in which a researcher has four identical, dichotomously scored indicators (A_h, B_j, C_l, D_m) collected in two samples (N_s, where S = 2). We may state this parameterization of the LCM as

$$\ln\left(\hat{f}_{hjlmgs}\right) = \lambda + \lambda_h^A + \lambda_j^B + \lambda_l^C + \lambda_m^D + \lambda_g^X + \lambda_s^N + \lambda_{hg}^{AX} + \lambda_{jg}^{BX} + \lambda_{lg}^{CX} + \lambda_{mg}^{DX} + \lambda_{gs}^{XN} + \lambda_{hs}^{AN} + \lambda_{js}^{BN} + \lambda_{ls}^{CN} + \lambda_{ms}^{DN} + \lambda_{hgs}^{AXN} + \lambda_{jgs}^{BXN} + \lambda_{lgs}^{CXN} + \lambda_{mgs}^{DXN} \quad (2)$$

where $\hat{f}_{hjlmgs} = \pi_{g|s} \prod_{i=1}^{k} \pi_{ix|gs} \times n$ 5with n representing the total sample size ($N_1 + N_2$). As in the usual loglinear model, the loglinear parameterization of the LCM is made identifiable by imposing the restriction that the lambda parameters sum to zero, for example:

$$\sum\nolimits_h \lambda_h^A = \sum\nolimits_h \lambda_{hg}^{AX} = \sum\nolimits_g \lambda_{hg}^{AX} = \sum\nolimits_h \lambda_{hs}^{AN} = \sum\nolimits_s \lambda_{hs}^{AN} = \sum\nolimits_h \lambda_{hgs}^{AXN} = \sum\nolimits_g \lambda_{hgs}^{AXN} = \sum\nolimits_s \lambda_{hgs}^{AXN} = 0\,. \quad (3)$$

Moreover, it is important to note that, as in the loglinear parameterization for the single-group LCM, in the multi-sample LCM the axiom of local independence restricts the indicator variables to have partial zero associations among each other; however, the grouping variable (N) is allowed to have non-zero associations with both the indicators and the latent variable(s).

In the loglinear parameterization of the LCM, we may also discuss model invariance in terms of structural and distributional homogeneity, albeit in a slightly different manner. Model (2) represents a loglinear model denoted as {AXN,BXN,CXN,DXN} in the standard short-hand notation for hierarchical loglinear models, in which all parameters may assume different values for the two groups, which amounts to employing a separate LCM for each group. If model {AXN, BXN,CXN,DXN} fits the data, we may conclude that in both groups a similar LCM is applicable, in the sense that the same number of latent classes can be used in both groups, but that otherwise there is complete model invariance. This is a case of complete measurement (and distributional) heterogeneity. In terms of the probabilistic parameterization: all of the probabilistic parameters are different for the two populations.

At the other extreme is the model implying complete measurement homogeneity where the grouping variable only influences the distribution of the latent variable X, that is, model {AX,BX,CX,DX,XN} in which all conditional response probabilities are the same for the two populations. The choice between the two extreme models can be made, as before, using conditional L^2 tests.

If the restrictive model {AX,BX,CX,DX,XN} has to be rejected, it is not always necessary to accept the least restrictive model {AXN,BXN,CXN,DXN}. There is an interesting „in-between" model {AX,BX,CX,DX,XN,AN,BN,CN,DN} which comes natural in the loglinear, but not in the probabilistic parameterization (Hagenaars, 1990, p. 127-135). In this „in-between" model, it is allowed that, due to the loglinear two-variable effects AN, BN, CN, and DN the *level* of the conditional probabilities (i.e., their „difficulties") may be higher or lower in one population compared to the other. However, the loglinear two-variable effects AX,BX,CX, and DX, that is, the *odds ratios* relating the indicator and latent variables remain constant. Given that, as in factor analysis, also in LCMs the meaning and interpretation of the latent variable to a large extent has to be derived from the relations (the „loadings“) between the indicators and the latent variable, these identical relationships make the comparative interpretation of X easier than in the invariance model {AXN,BXN,CXN,DXN}.

Once the researcher is satisfied with and can give a meaningful interpretations to the measurement homogeneities and heterogeneities found, a test on the distributional homogeneity is possible by considering models in which there is no relation between the grouping variable N and the latent variable X. In the most restrictive model discussed above {AX,BX,CX,DX,XN} the term XN is replaced by N, leading to model {AX,BX,CX,DX,N}. Independence of X and N is a bit more difficult to impose in the other two models, because, given the structure of the models with direct effects of N *and* X on the indicator variables, a modified path (or LISREL) approach has to be followed (Goodman, 1973; Hagenaars 1990, 1993). Imposing independence between X and N in the least restrictive model {AXN,BXN,CXN,DXN} or in the „in-between“ model {AX,BX,CX,DX,XN,AN,BN,-CN,DN} in both cases requires first imposing submodel {X,N} on the marginal „frequency“ table {XN} followed either by submodel {AXN,BXN,CXN,DXN} or {AX,BX, CX,DX,-XN,AN,BN,CN,DN} for the complete „frequency“ table ABCDXN; finally, the estimated frequencies for the two submodels have to be combined in the usual 'modified path' manner to obtain the estimates for the complete model. The usual conditional L^2 tests can be used to decide among the several models.

3. Probabilistic and Loglinear Model Equivalences

The unrestricted, multi-sample LCM represented in equations (1) and (2) are essentially equivalent, requiring the estimation of an identical number of parameters and netting identical expected values; as a result, these models have identical chi-square values with an identical number of degrees of freedom for the model test. Although, as we saw above, the loglinear parameterization leads *naturally* to a somewhat different kind of substantively interesting parameter restrictions than the probabilistic parameterizations, exactly because of the correspondence between the two parameterizations we will in this section exemplify how the different kinds of restrictions can be translated into each other.

As many authors have noted, both for the single-sample and the multi-sample LCM, there is an equivalence between conditional probabilities of the probabilistic parameterization and the lambda coefficients of the loglinear parameterization, where the exact nature of the correspondence also depends on whether dummy coding or effect coding has been used for the loglinear parameters (Haberman, 1979, p. 551; McCutcheon, 1994; Hagenaars, 1990; Heinen, 1993). (In this chapter, only effect coded loglinear parameters are discussed). By way of example, the conditional probability of responding at level 1 of the second indicator (B_1) given that one is at level 2 of the latent variable, as expressed in (36), can also be expressed as a ratio of the loglinear coefficients:

$$\pi_{21|gs} = \frac{\exp\left(\lambda_1^{B} + \lambda_{1g}^{BX} + \lambda_{1s}^{BN} + \lambda_{1gs}^{BXN}\right)}{\sum_j \exp\left(\lambda_j^{B} + \lambda_{jg}^{BX} + \lambda_{js}^{BN} + \lambda_{jgs}^{BXN}\right)} \; . \tag{4}$$

In the previous section, it was seen that model {AX,BX,CX,DX,XN} implies that the conditional response probabilities are the same for both populations. In agreement with this, it follows from inspection of (4) that when it is possible to restrict to zero both of the parameters which include the indicator and grouping variables $(\text{e.g.}, \lambda_{js}^{BN} = \lambda_{jgs}^{BXN} = 0)$, the resulting loglinear LCM is equivalent to the probabilistic LCM in which it the specified indicator variable 'behaves identically' across groups (i.e.,$\pi_{ix|g1} = \pi_{ix|g2}$).

From (4) it also follows that making two indicator variables (e.g., A and B) 'parallel' measures of the latent variable, in the sense that $\pi_{11|11} = \pi_{21|11}$6 is equivalent to imposing a set of restrictions such as $(\lambda_1^A + \lambda_{11}^{AX} + \lambda_{11}^{AG} + \lambda_{111}^{AXG}) = (\lambda_1^B + \lambda_{11}^{BX} + \lambda_{11}^{BG} + \lambda_{111}^{BXG})$ 7 in the loglinear parameterization.

At other times, the investigator may be interested in testing *equal error rates* hypotheses for the indicator variables. The researcher then views the indicator variables as being subject to error in the classification of observations. In our earlier example, for instance, we may view each of the four indicator items as underlying measures of the respondents' „true“ (latent) attitude toward legal demonstrations. Consequently, among the Dutch „approver“ (Class I) respondents there is a (1.000-0.872) 0.128 probability of providing a false („would not participate“) response, whereas among the Dutch „disapprover“ (Class 2) respondents there is a .152 probability of giving a false („would participate“) response; in epidemiological and educational research these are commonly referred to as *false positives* and *false negatives*. To test the equal error rates hypothesis in the probabilistic parameterization of the multi-sample model, equality restrictions are imposed on the corresponding conditional probabilities within a group (e.g., $\pi_{11|11} = \pi_{12|21}$8). In the loglinear parameterization of the multi-sample model, equality restrictions are placed on the lambda parameters of (4) which do not include the latent variable $(\text{e.g.}, \lambda_1^A = \lambda_{11}^{AG} = 0)$. Finally, we can test this hypothesis across-groups by combining the parallel across group restrictions with the equal error rates test9 $(\text{e.g.}, \lambda_1^A = \lambda_{11}^{AG} = \lambda_{111}^{AXG} = 0)$.

In this way, the more common hypotheses in the multi-sample LCM can be expressed readily by either parameterization. Moreover, computer software is now available to use both parameterizations (although sometimes, as Mooijaart and van der Heijden (1992) note, we must exercise caution when using certain kinds of 'asymmetrical' equality constraints on the conditional response probabilities (e.g., $\pi_{11|11} = \pi_{11|12} = \pi_{21|11}$10).,because in those cases the standard algorithms in the latent class programs may yield wrong estimates.

4. Example

The joint distribution of the responses to the four protest approval items is presented in Table 2. The extra insights that LCM may provide can be simply illustrated by noting the observed fact that these data indicate that Dutch respondents (N = 729) appear to be more positive toward student demonstrators (66%) than do the American respondents (35%, N = 870). As the LCM outcomes presented in Table 1 indicates, however, this higher level of positive feelings toward student demonstrators may be the result of an overall more favorable disposition toward legal demonstrations among the Dutch; Dutch and American „approvers“ (Class I) may be similarly disposed toward student demonstrators, and the higher frequency of positive sentiment toward student demonstrators among the Dutch sample may be the result of a higher incidence of Class I respondents in the Netherlands. The programs MLLSA

(Clogg, 1977) and LEM (Vermunt, 1993, 1996) are used to estimate the probabilistic and loglinear LCMs, respectively.

			Dutch		*American*	
			D.Feelings Toward Student Demonstrators			
			Positive	*Negative*	*Positive*	*Negative*
C.Law Opposing	B.Participation	A.Approval				
Oppose	Would	Approve	365	140	237	297
		Disapprove	11	9	9	28
	Would Not	Approve	54	34	16	30
		Disapprove	14	25	10	39
Favor	Would	Approve	14	9	13	42
		Disapprove	3	3	2	25
	Would Not	Approve	5	7	5	23
		Disapprove	4	22	14	80

Table 2: Observed crosstabulation for support of public demonstrations: 1972 American and Dutch samples

First, we will analyze these data mainly from the perspective of the probabilistic parameterization. In Table 3, we report the likelihood-ratio (L^2) and goodness-of-fit (X^2) chi-square statistics for some of the possible LCMs for these data. As we see, the unrestricted LCM H_1,(in loglinear terms model {AXN,BXN,CXN,DXN}), the outcomes of which are presented in Tables 1 and 3) is an acceptable characterization of the joint distribution presented in Table 2 ($L^2 = 13.22$, $X^2 = 12.71$, df. = 12, p > .30). In both populations a 'comparable' two latent class solution is possible. The solutions are not completely comparable, however, as we cannot accept the preferred complete structural homogenenity hypothesis H_2 (corresponding to loglinear model {AX,BX,CX,DX,XN}), since the conditional L^2 test for this model is unacceptably large relative to the degrees of freedom (172.65–13.22 = 159.43, 20–12 = 8 df.).

Inspection of the estimates of the (conditional) probabilities of the heterogeneous model H_1, reported in Table 1, suggest the partial invariance model H_2, that lies somewhere between the models H_1 and H_2. Equality restrictions are imposed on parameter estimates that are rather close to each other. The results are reported in Tables 3 and 4. Two sets of equality constraints are imposed. The first of these is an across-nation equality restriction on the approval of legal demonstrations indicator item A. As we see from the reported conditional probabilities in Table 4, Class I respondents in both the Netherlands and the United States exhibit a .971 chance of approving of legal demonstrations, while Class II respondents in both nation exhibit a .231 chance of approval for this indicator.

Model	L^2	X^2	*DF*
H_1 : Heterogeneous 2-Class	13.22	12.71	12
H_2 : Complete Structural Homogeneity	172.65	176.73	20
H_3 : Partial Structural Homogeneity	13.70	13.20	15
H_4 : Distributional Homogeneity for Model H_3	42.01	41.18	16

Table 3: Likelihood-ratio and goodness-of-fit chi-squares for the probabilistic parameterization of the multi-sample latent class model

The second set of equality restrictions that we have imposed indicates that the participation item B has an *equal error rate* for the Dutch population. Among the „approvers" (Class I), the probability of a respondent reporting that he or she would participate is .870, while among the „disapprovers" (Class II), respondents report a .130 probability of approving; thus, the Dutch „approvers" and „disapprovers" exhibit an identical error rate with respect to the participation item (.130).

This restricted, partial measurement homogeneity model (H_3) fits the data well, according to the test statistics reported in Table 3 (with $p > .50$), and has to be preferred to the heterogeneous model according to the conditional test statistic: $L^2 = 13.70 - 13.22 = 0.48$, with df = 15 – 12 = 3. Here, however, we must take into account that this is a case of post hoc testing; after inspecting the outcomes of the Heterogeneous Model (H_1), we restricted selected parameters, forwarding no theoretical reasons to justify these particular restrictions.

Having accepted Model H_3, the distributional homogeneity hypothesis can be tested by imposing the extra restriction that the relative distribution of the latent variable is the same in the Netherlands and the USA, i.e., it is assumed that in marginal table NX, N and X are independent of each other. Looking at the test outcomes for H_4, it is clear that this independence hypothesis has to be rejected ($L^2 = 42.01 - 13.70 = 28.31$, 16 – 15 = 1 df.). Thus, we must conclude that the Dutch are significantly more likely than Americans to approve of legal demonstrations (.850 versus .720 - see Table 4).

	Dutch		*American*	
Indicator Item	Class I	Class II	Class I	Class II
A. Approval (Approve)	.971[a]	.231[a]	.971[a]	.231[a]
B. Participation (Would)	.870[b]	.130[b]	.945	.252
C. Law (Oppose)	.960	.521	.924	.359
D. Student Demonstrators (Positive)	.712	.354	.432	.145
Latent Class Probabilities	.850	.150	.720	.280

[a] Across-group equality restrictions. [b] Within-group equal error rate restrictions.

Table 4: Conditional and latent class probabilities for model H_3

Now turning to the loglinear parameterization, as we noted earlier, the unrestricted LCM is the same for the two parameterizations, as a comparison of H_1 in Table 3 and Table 5 indicate. At the other extreme is the complete structural homogeneity model H_2, which is also identical in the two parameterizations.

Model H_3 is the loglinear „in-between" model dealt with before (but not identical to model H_3 in Table 3). It fits the data well ($p > .60$). Imposing upon this model the extra restriction that in the marginal table X and N are independent of each other yields model H_4 which barely fits the data ($p = .056$), but significantly worse than model H_3 ($L^2 = 27.15 - 13.65 = 13.50$, df = 17 – 16 = 1, $p < .001$).

Model	L^2	X^2	*DF*
H_1 : Heterogeneous 2-Class {AXN,BXN,CXN,DXN}	13.22	12.71	12
H_2 : Complete Structural Homogeneity {AX,BX,CX,DX,NX}	172.65	176.73	20
H_3 : Partial Structural Homogeneity {AX,BX,CX,DX,XN,AN,BN,CN,DN}	13.65	13.13	16
H_4 : H_3 + Distributional Homogeneity {X,N}{AX,BX,CX,DX,XN,AN,BN,CN,DN}	27.15	24.86	17

Table 5: Likelihood ratio and goodness of fit chi-squares for the loglinear parameterization of the multi-sample latent class model

The loglinear parameter estimates for H_3 in Table 5 have been reported in Table 6. From the second row in the lower panel of this latter table, it is clear that all indicators are positively correlated with the latent variables (net of the influence of N). So, class I of X denotes the approvers of protest and class II the disapprovers. A has the strongest relationship with X (is the most reliable indicator), then B, then C, and finally, D, which appears to be a poor indicator (according to this model).

	for table ABCDXN	*for table XN*
λ	1.416	2.995
λ_1^X 1 1	-.251	.668
λ_1^N 1 2	-.198	-.196
λ_{11}^{XN} 13	.114	.186

k:	*A*	*B*	*C*	*D*
λ_1^k 14	.575	.212	.648	-.218
λ_{11}^{kX} 15	1.161	.965	.772	.377
λ_{11}^{kN}	.040	-.206	.180	.298

Table 6: Loglinear parameter estimates for model H_3 -Table 5

The last row of the lower panel in Table 6 renders the partial associations of the indicators and the grouping variable N, net of the effects of the latent variable X. Other than the differences caused by the different scores on the latent variable, there are no differences between the people in the Netherlands and the USA with regard to their approval of demonstrations. However, the Dutch are relatively more inclined to oppose laws restricting demonstrations and feel relatively more positive about student demonstrators, while the Americans are more willing to participate in demonstrations. As far as these relationships reflect national and historic differences that as such have nothing to do with the underlying (latent) attitude „Approval of Protest“ the „in-between“ model takes appropriately care of what would otherwise bias the measurement results.

Once this measurement model is accepted, it makes sense to investigate the national differences with regard to the latent variable Approval of protest. The relevant parameter denoting the relation between N and X is presented in the last row of the upper panel of Table 6. For illustration, two values have been reported; however, only the last one is of substantive interest, as the coefficient in the first column is computed holding the indicators A,B,C, and D constant, which given the causal order of the variables does not make sense. It then appears that there is a weak positive relation between X and N, meaning that the Dutch are somewhat more likely than Americans to approve protests.

Given the outcomes in Table 6, it is of course possible to find still more parsimonious models. One might, for example, restrict the relation between N and A to being exactly zero. Or, A and B could be made into parallel indicators by making their relationship with X exactly the same. It is also possible to translate the restrictions of the probabilistic parameterization into restrictions on the loglinear parameters, and to combine some of the results of Tables 4 and 6. Finally, the conditional response probabilities and the latent probabilities may be computed, using either the loglinear parameters or, which is certainly easier to do, using the estimated expected frequencies of the loglinear model.

5. Conclusion

The latent class model has found increasing use as an approach to exploring categorically-scored data. As we have seen here, the multi-sample LCM offers a powerful and flexible approach for comparative social research. The two parameterizations offer complementary approaches, each leading to somewhat different sets of concerns. Although it is clear that model restrictions with one parameterization can be replicated in the other, the complementary advantages may be such that the practicing researcher may wish to explore both parameterizations.

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