Chapter 26

An Application of a Rasch-Based Unfolding Model to a Questionnaire on Adolescent Centrism

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The development of item response models with single-peaked item characteristic curves (ICC), termed *unfolding models* in the literature (Coombs, 1964), has found a renewed interest for attitude measurement recently. Several authors have made substantial progress in formulating such „unfolding“ models for dichotomous item response data, either on the basis of Rasch measurement theory (Andrich, 1988; Andrich and Luo, 1993; Hoijtink, 1990; Verhelst and Verstralen, 1993), latent class analysis (Böckenholt, 1993; Croon 1993; Formann, 1988), or non-parametric item response theory (Post and Snijders, 1993; v.Schuur 1993).

Compared to the broad application of the cumulative modelling theory, the kernel of which is the Rasch model, however, the unfolding models are still far from being routinely applied in test analysis. One of the main perplexities of the parameterization of unfolding models is the inability to maintain the principle of *specific objectivity*, which is taken as an axiom in Rasch measurement theory. In addition, because of the lack of appreciation of unfolding models, many item response data invoked in the field of attitude measurement were improperly analyzed using monotonic-ICC models.

The analysis of an attitude questionnaire presented in this chapter illustrates the differences of analyzing attitude items with a cumulative and an unfolding model, respectively. Before presenting these results, a specification of the *general hyperbolic cosine model* (eq. (26) in chapter 1) is introduced.

1. A specification of the general hyperbolic cosine model with equidistant thresholds

In the model derived by Andrich (1996), i.e., model (26) in chapter 1, the threshold distances are not restricted at all, i.e., they may vary among the categories within each item as well as among the items. The basis of model (26) is the unrestricted Rasch model for ordinal data, i.e., model (14) in chapter 1.

When working with rating scales, it often makes sense to assume that the thresholds are equidistant, i.e., the differences of adjacent thresholds are constant for all categories of an item:

$$\tau_{ix} - \tau_{i(x-1)} = 2\,\delta_i \quad \text{for} \quad 1 < x \le m .$$

Imposing this restriction on the polytomous Rasch model (14, chapter 1) results in the *equidistant Rasch model* developed by Andrich (1982, (see eq. (18) in chapter 1)):

$$p\left(X_{vi}=x\right)=\frac{1}{d_{vi}}\exp\left(x\theta_v - x\sigma_i + x\left(m-x\right)\delta_i\right), \tag{1}$$

where σ_i represents the mean of all thresholds of item i, δ_i is half the distance of two adjacent thresholds, and d_{vi} is a normalizing factor, i.e., the sum of the numerator over all categories.

The motivation for deriving the unfolding model under the same restriction may not only be seen in the reduction of the number of parameters from m-1 independent parameters to a single one per item, but primarily in the interpretation of this item parameter. As it will become evident later, it is a kind of dispersion parameter that governs the width and the altitude of the item response function. Moreover, this distance parameter represents the direct generalization of what has been called the *unit parameter* in the dichotomous hyperbolic cosine model (Andrich and Luo, 1993; Luo, 1994; Andrich, 1996).

Last but not least, an equidistance assumption, although a weaker one, also has to be made in the general unfolding model by Andrich (1996, see (26) in chapter 1). When this model is derived by collapsing the symmetric categories of an extended rating scale with $2m+1$ categories (see chapter 1), it has to be assumed that each pair of joined categories has equally spaced thresholds (see Andrich, 1996).

Adding the response probabilities defined by the equidistant Rasch model (1) for the (hypothetical) extended rating scale with $2m+1$ response categories gives the following probabilities for the observed response variable:

$$p\left(X_{vi}=x \middle| x<m\right)=\frac{1}{d_{vi}}\cdot$$
$$\left[\exp\left(x\theta_v - x\sigma_i + x\left(2m-x\right)\delta_i\right)+\exp\left(\left(2m-x\right)\theta_v-\left(2m-x\right)\sigma_i+\left(2m-x\right)\left(2m-2m+x\right)\delta_i\right)\right],$$

where the coefficient of the second δ_i-parameter reduces to that of the first, i.e., $\left(2m-x\right)x$. As a consequence, this equation can be rewritten as

$$\begin{aligned} p\left(X_{vi}=x \middle| x<m\right) &= \frac{1}{d_{vi}}\left[\exp\left(x(2m-x)\delta_i\right)\left(\exp\left(x\left(\theta_v-\sigma_i\right)\right)+\exp\left(\left(2m-x\right)\left(\theta_v-\sigma_i\right)\right)\right)\right] \\ &= \frac{1}{d_{vi}}\left[\exp\left(x\left(2m-x\right)\delta_i\right)\exp\left(m\left(\theta_v-\sigma_i\right)\right)2\cosh\left(\left(m-x\right)\left(\theta_v-\sigma_i\right)\right)\right], \end{aligned} \tag{2}$$

where cosh is the hyperbolic cosine function cosh (x) = (exp(x) + exp(-x))/2. The probability for the highest response category, m, of the folded rating scale is not a sum of two joined categories, but is simply defined by model (1) as the probability of the *middle* category of the extended rating scale:

$$p\left(X_{vi}=m\right)=\frac{1}{d_{vi}}\exp\left(m\left(\theta_v-\sigma_i\right)+m^2\,\delta_i\right). \tag{3}$$

Since the exponential function $\exp(m(\theta_v - \sigma_i))$ is a constant factor of all numerators in (2) and (3), it cancels and the model equation reduces to

$$p\left(X_{vi} = x \middle| x < m\right) = \frac{1}{d_{vi}}\left[\exp\left(x\left(2m - x\right)\delta_i\right) 2 \cosh\left(\left(m - x\right)\left(\theta_v - \sigma_i\right)\right)\right], \quad \text{and}$$
$$p\left(X_{vi} = m\right) = \frac{1}{d_{vi}} \exp\left(m^2 \delta_i\right). \tag{4}$$

Figure 1 shows the category response functions of model (4) for an item with 4 response categories.

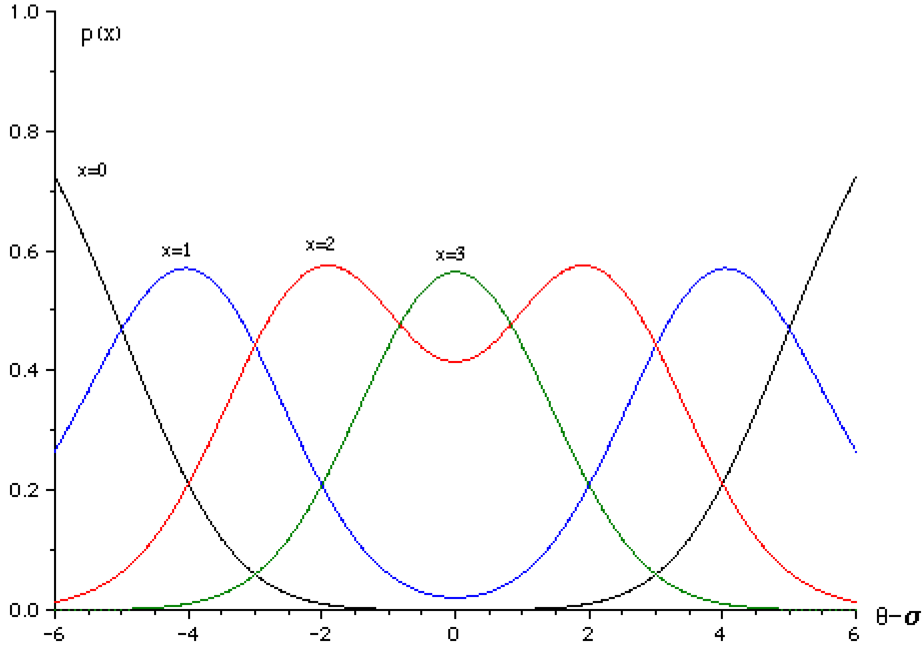

Figure 1: The category response functions of an item with 4 categories according to model (4)

In order to verify that the *item response function* of this model is *single peaked*, whereas that of model (1) is monotonous, Figure 2 shows the expected item response as a function of the latent trait θ according to both models. The expected item response is defined as

$$E\left(X\right) = \sum_{x=0}^{m} x\, p\left(X = x\right) \tag{5}$$

and its functional dependency on the trait can be viewed as the generalization of the concept of the IRF to more than two categories (it is noted that in the dichotomous case E(X) = p(X = 1)).

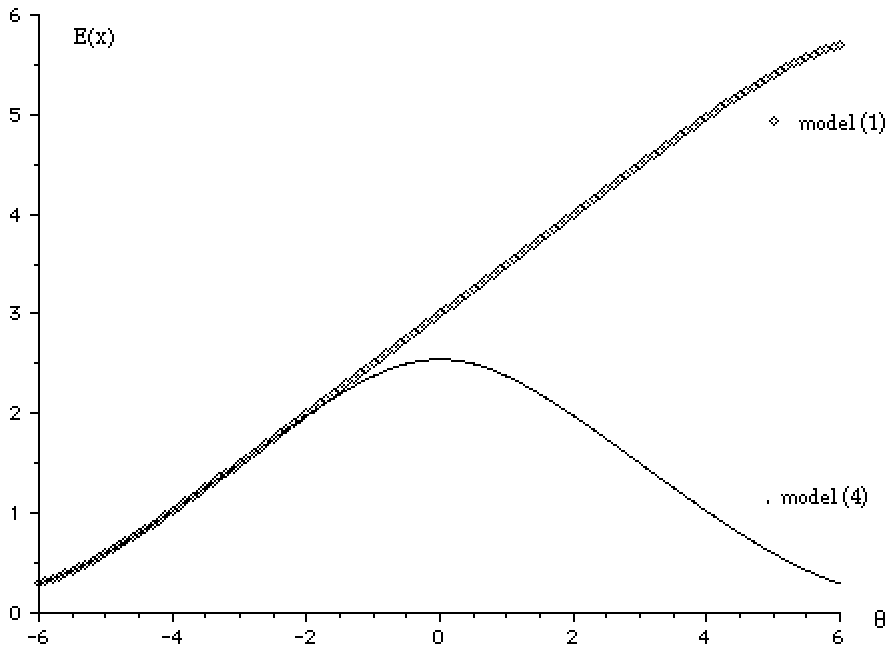

Figure 2: The expected item response as a function of θ for models (1) and (4), respectively

As in the dichotomous case, the distinction between monotonous and single-peaked IRF corresponds to the distinction between a cumulative and an unfolding response process in the case of polytomous item responses.

The parameter value δ_i determines the steepness and width of the item response function of model (4). Figure 3 demonstrates that, for different values of δ, the response functions do not intersect. That is, for any fixed value of θ, the expectation of the item response increases as the value of δ increases.

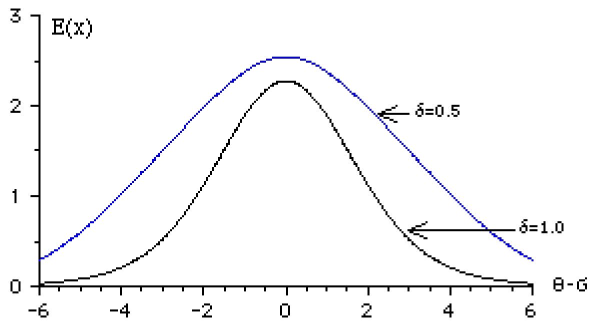

Figure 3: The effect of item parameter δ on the shape of the item response function

As in the cumulative model (1), the item parameter δ governs the dispersion of the item responses across the categories and, hence, is called the *dispersion parameter* (cf. Andrich, 1982).

Parameter estimation of the polytomous unfolding model (4) turns out to be rather difficult, due to the lack of simple sufficient statistics for parameters, as Andrich and Luo (1993) show for the dichotomous case of model (4). However, the observed data of model (4) can be considered as incomplete data (due to collapsing symmetric categories of the extended rating scale) while the complete data fit the ordinal Rasch model. As is well known, the maximum likelihood (ML) estimation for ordinal Rasch models is efficient, so that the EM-algorithm (Dempster, Laird and Rubin, 1977) can be applied for the parameter estimation of the polytomous unfolding model.

Luo and Rost (1994) applied a joint ML-estimation procedure in the M-step of the EM-algorithm, which turned out to let the EM-algorithm diverge when the dispersion and difficulty parameters of the items were estimated simultaneously. The results of the application described below were obtained by a two-step procedure, estimating the dispersion parameters first, and turned out to be very stable: they were checked for different starting points of the iteration procedure as well as for different subsets of items and persons. After finishing this paper we came to know that Roberts and Laughlin (1996) have written a computer program for a strongly related model.

2. A questionnaire on adolescent centrism

A ten-item questionnaire aimed at measuring the attitude of young people towards the world of adults, called „adolescent centrism“, was analyzed with the polytomous unfolding model. The data were gathered as part of a large scale study in Germany (Jugendwerk Deutsche Shell 1985) in which a total of 1472 adolescents responded to this questionnaire. The items are shown in Table 1.

1.	The police force treats adolescents unfairly
2.	In our society you are confronted with animosity everywhere - that is completely demoralizing
3.	Our society really does a lot for young people
4.	Young people put their foot down - when necessary - and do not put up with everything at work
5.	I really owe a lot to my parents
6.	I try to understand my parents - even when it is difficult
7.	Very few adults really understand young people's problems
8.	I don't believe in adults' experience - I prefer to depend on myself
9.	I learn more from friends my own age than from my parents
10.	Parents always interfere in things that are none of their business

Table 1: The items of the questionnaire on adolescent centrism

These items have already been analyzed with a cumulative Rasch model and its generalization to a mixture distribution model, the *mixed Rasch model* (Rost 1990, 1991). In these analyses it turned out that about 80 % of the sample (class 1) fit the Rasch model, whereas the second latent class, covering about 20 %, can be interpreted as a class of *unscalables* (Rost and Georg, 1991). This result makes sense, because in such kinds of large scale field studies a considerable percentage of respondents handle the questionnaire in a slipshod fashion, resulting in more or less noisy data.

Nevertheless, we analyzed the whole data set using the unfolding model, because any selection of „unscalable“ persons according to the previous results would be based on the mixed Rasch analysis and, hence, on the assumption that the response behavior of the „scalables“ follow the cumulative model.

Special attention is paid to the three items located at the left end of the adolescent centrism scale, i.e., item 3 („Our society does a lot for young people“), item 5 („I really owe a lot to my parents“), and item 6 („I try to understand my parents - even when it is difficult“). From the perspective of the cumulative model, i.e., in the tradition of Likert-scaling, these items are formulated in the opposite direction (negatively verbalized) and have to be recoded accordingly. Whereas the other items are coded from '0' („disagree“) to '3' („agree“), for these three items the response category '3' means „disagree“ and '0' means „agree“.

From the perspective of the unfolding model, i.e., in the tradition of Thurstone-scaling, these items are simply located at the left hand side of the attidude dimension, directed towards the opposite pole of „adolescent centrism“, which may be described as „openess towards the adults world“. For the application of an unfolding model, these items do not have to be recoded.

Since the EM-algorithm developed by Luo and Rost (1994) requires estimates of the dispersion parameters δ_i (cf. eq. (4)) for estimating the location parameters, the former have been estimated by means of a latent class model (Rost 1988) and the Rasch model for ordinal data, both with equidistant thresholds. The results turned out rather stable for the 3- and 4-class solutions of the latent class model and the Rasch model (cf. Table 2).

item number	*latent class models*		*Rasch model*
	3 classes	*4 classes*	
1	.40	.49	.41
2	.63	.71	.61
3	.73	.71	.73
4	.36	.36	.41
5	.34	.32	.30
6	.41	.41	.52
7	.59	.59	.63
8	.46	.49	.44
9	.49	.52	.47
10	.55	.58	.53

Table 2: Estimates of the dispersion parameters of two latent class models and the ordinal Rasch model

These estimates were used for estimating the locations of the items according to the unfolding model. In order to gain some knowledge about the stability of the solution, the parameters were estimated in two equally sized subsamples. Additionally, all analyses were done with different sets of initial values to ensure that the global maximum has been found. Table 3 gives the estimates of the location parameters of the unfolding model in both sub-samples as well as the location parameters of the Rasch model.

item	*unfolding model*		*Rasch model*
number	*sample 1*	*sample 2*	
1	1.19	1.25	0.31
2	2.07	2.07	0.46
3	-2.97	-3.00	-0.29
4	-0.04	-0.04	-0.53

5	-0.96	-1.01	1.16
6	-0.87	-0.95	1.28
7	0.10	0.04	-1.06
8	0.44	0.53	-0.37
9	0.47	0.51	-0.44
10	0.55	0.60	-0.53

Table 3: Estimates of the location parameters of the unfolding and the Rasch model

In order to facilitate the comparison of both sets of parameters, Figure 4 shows the parameter profiles of both models. In this figure, the items are ordered along the abscissa according to their location parameter in the unfolding model.

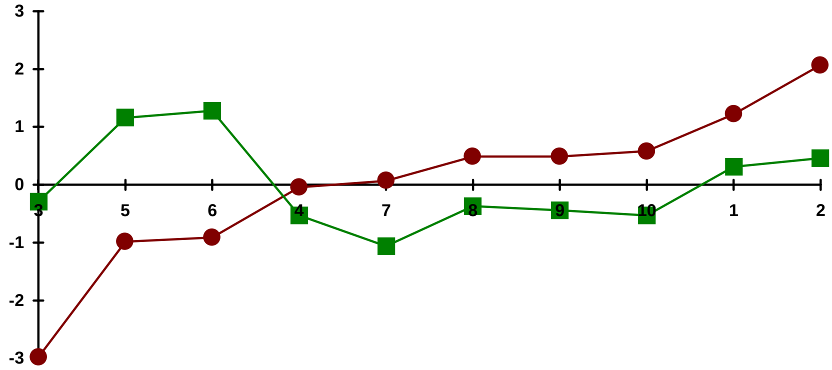

Figure 4: The profiles of the location (difficulty) parameters of the unfolding (dots) and the Rasch model (squares). The item numbers are depicted on the abscissa.

It turns out that the profiles run quite parallel for the 7 items formulated in the positive direction and that a cross-over is given for the three negative items 3, 5, and 6. At first glance, the fact that the order of the three negative items is preserved when moving from one to the other model, seems to contradict intuition. Thinking in terms of *folding* an unfolded scale, intuitively leads to the expectation that the order of the item locations is reversed when recoding the items, as is done with the negative ones.

However, a closer look exposes the incorrectness of this expectation: item 3, which is far left on the unfolding scale, requires a very low adolescent centrism of a person in order to *agree* with it („...society does a lot ...“). After recoding the item to apply the cumulative model, a relatively low degree of adolescent centrism is sufficient to *disagree* with that item. As compared with item 3, it is harder, i.e., requires more adolescent centrism, to *disagree* with items 5 and 6, which are located right of item 3 (on the θ-scale) in both models.

Hence, Figure 4 shows what typically happens when analyzing attitude items with the Likert technique: recoding the negative items and applying a cumulative model causes a *shift* (not a turn-over!) of the negative items into the area of the positive items on the latent continuum. The metaphor of „unfolding a joint scale“ and „folding an unfolded scale“ does not hold for the situation where negatively formulated items are recoded and jointly analyzed with a cumulative model.

Another conclusion is related to this finding. Whereas the difficulty parameters of the unfolding model can be compared among *all* items, an interpretation of the parameters of negative (and recoded) items in relation to those of the positive items can hardly be done in the cumulative model. As a result of recoding the responses and assuming a cumulative response process, the relations among the negative items may be preserved, but the relations to the locations of the positive items change in a rather obscure way. Apart from that, what does 'it is more difficult to agree with item i than to disagree with item j' mean?

3. Discussion

There are a lot of unanswered questions regarding the analysis of attitude questionnaires with unfolding and cumulative models. It was not the aim of the present application to decide which the two models fits the data better. Rather it was aimed at showing what happens when the same data set is analyzed with these two different kinds of models. The results fit with what is expected when a questionnaire with single peaked response functions is analyzed by an IRT-model with monotonous response functions. Results gained with simulated data show the same pattern of parameters under both models. This gives some indirect evidence for the superiority of an unfolding IRT-model in the case of this questionnaire on adolescent centrism.

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References

Andrich, D. (1982). An extension of the Rasch model for ratings providing both location and dispersion parameters. *Psychometrika*, *47, 1*, 105-113.

Andrich, D. (1988). The Applications of an unfolding model of the PIRT Type to the measurement of attitude. *Applied Psychological Measurement*, *12*, 33-51.

Andrich, D. (1996). A hyperbolic cosine latent trait model for unfolding polychotomous responses: Reconciling Thurstone and Likert methodologies. *British Journal of Mathematical and Statistical Psychology* (in press).

Andrich, D. & Luo, G. (1993). A hyperbolic cosine latent trait model for unfolding dichotomous single-stimulus responses. *Applied Psychological Measurement*, *17*, 253-276.

Böckenholt, U. (1993). Analysis of 'pick any/n' data with no prior knowledge: A synthesis of latent class analysis and unfolding models. *Kwantitatieve Methoden, 42,* 109-126.

Coombs, C.H. (1964). *A theory of data*. New York: Wiley (2nd ed. 1976).

Croon, M. (1993). Ordinal latent class analysis for single-peaked items. *Kwantitatieve Methoden, 42,* 127-142.

Dempster, A.P., Laird, N.M., Rubin, D.B. (1977). Maximum likelihood estimation from incomplete data via the EM-algorithm. *Journal of the Royal Statistical Society, 39,* 1-22.

Formann, A.K. (1988). Latent class models for nonmonotone dichotomous items. *Psychometrika, 53, 1*, 45-62.

Hoijtink, H. (1990). A latent trait model for dichotomous choice data. *Psychometrika*, *55*, 641-656.

Jugendwerk der Deutschen Shell (1985). *Jugendliche und Erwachsene '85. Generationen im Vergleich*. Opladen.

Luo, G. (1994). *The Hyperbolic Cosine Model: Models for unfolding and preference data*. Murdoch University, Perth.

Luo, G. & Rost, J. (1994). *POLYFOLD - A program for estimating the parameters of the Generalized Hyperbolic Cosine Model*. Murdoch University, Perth.

Post, W.J. & Snijders, T.A.B. (1993). Nonparametric unfolding models for dichotomous data. *Methodika, 7,* 130-156.

Roberts, J.S. & Laughlin, J.E. (1996). A unidimensional item response model for unfolding responses from a grade disagree-agree response scale. *Applied Psychological Measurement,* 1996, (in press).

Rost, J. (1988). Rating scale analysis with latent class models. *Psychometrika, 53, 3*, 327-348.

Rost, J. (1990). Rasch models in latent classes: An integration of two approaches to item analysis. *Applied Psychological Measurement, 14*, 3, 271-282.

Rost, J. (1991). A logistic mixture distribution model for polychotomous item responses. *The British Journal of Mathematical and Statistical Psychology, 44*, 75-92.

Rost, J. & Georg, W. (1991). Alternative Skalierungsmöglichkeiten zur klassischen Testtheorie am Beispiel der Skala "Jugendzentrismus". *Zentral-Archiv-Information*, *28*.

Van Schuur, W.H. (1993). Nonparametric unidimensional unfolding for multicategory data. *Political Analyis, 4*, 41-74.

Verhelst, N.D. & Verstralen, H.H.F.M. (1993). A stochastic unfolding model derived from the partial credit model. *Kwantitatieve Methoden*, *42*, 73-92.